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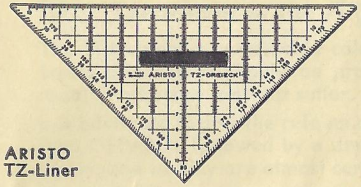

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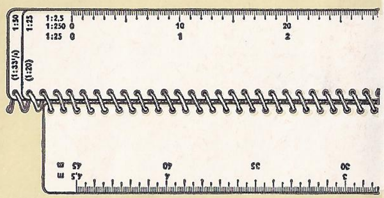

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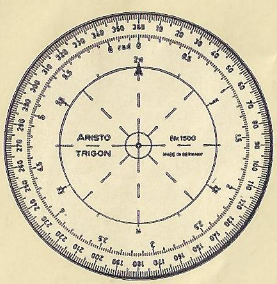

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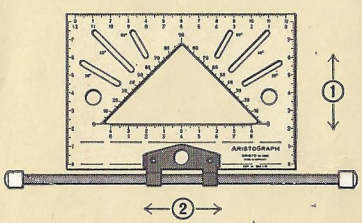

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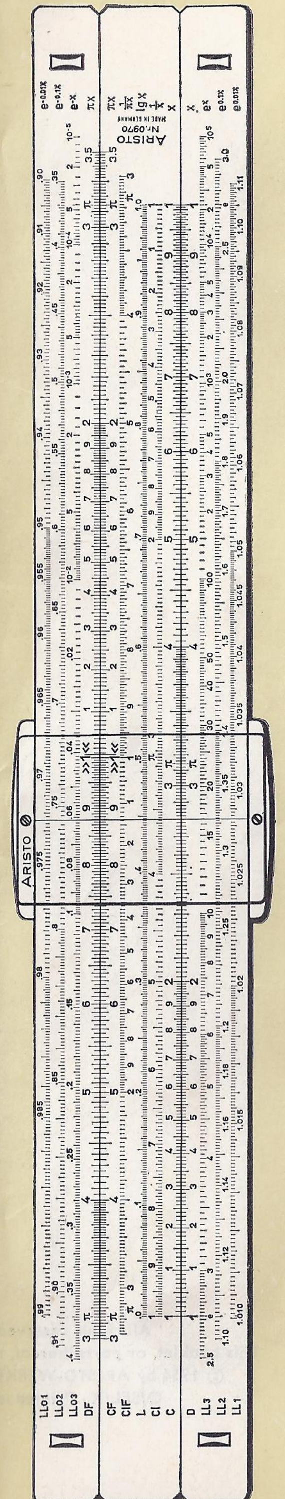

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CONTENTS

THE SLIDE RULE ARISTO MULTILOG

1. The Scales

The ARISTO MultiLog is a universal Log Log slide rule for scientists, engineers and students.

Front of Rule:

LL01	Log Log Scale, Range: .99—.9	$e^{-0.01x}$		Upper panel of body
LL02	.91—.35	$e^{-0.1x}$		
LL03	.4—.00001	e^{-x}		
DF	Folded Scale	πx		
CF	Folded Scale	πx		On slide
CIF	Reciprocal Scale folded by π	$1/\pi x$		
L	Mantissa Scale	lg x		
CI	Reciprocal Scale	$1/x$		
C	Fundamental Scale	x		
D	Fundamental Scale	x		Lower panel of body
LL3	Log Log Scale, Range: 2.5 to 100000	e^{x}		
LL2	1.1 to 3.0	$e^{0.1x}$		
LL1	1.01 to 1.11	$e^{0.01x}$		

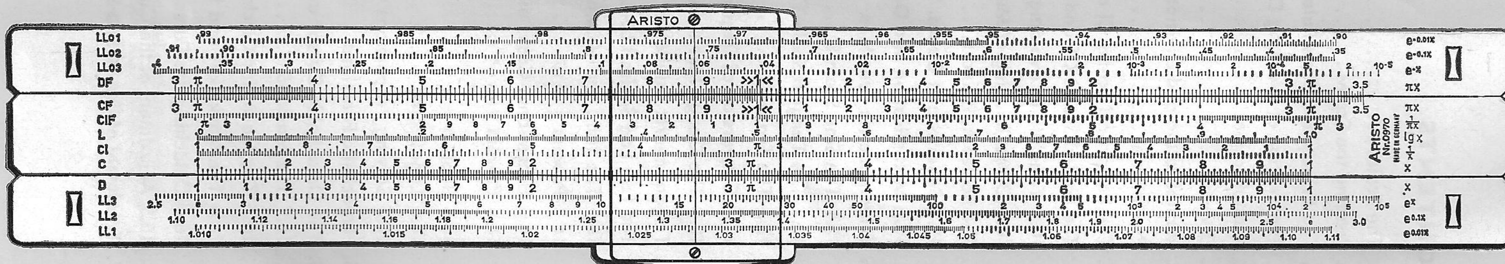

Fig. 1 Front of Rule

Back of Rule:

LL00	Log Log Scale, Range .999 to .989	$e^{-0.001x}$		Upper panel of body
K	Scale of Cubes	x^3		
A	Scale of Squares	x^2		
B	Scale of Squares	x^2		On slide
T	Scale of Tangents figured in black 5.5° to 45°; figured in red, counter-clockwise, 45° to 84.5° Also available for Cotangents	∢ tan		
ST	Scale of Small Angles, in radians, .55° to 6°	∢ arc		
S	Scale of Sines figured in black, 5.5° to 90°, figured in red, counter-clockwise for Cosines 0° to 84.5°	∢ sin		
C	Fundamental Scale	x		
D	Fundamental Scale	x		Lower panel of body
DI	Reciprocal Scale	$1/x$		
LL0	Log Log Scale, Range 1.001 to 1.011	$e^{0.001x}$		

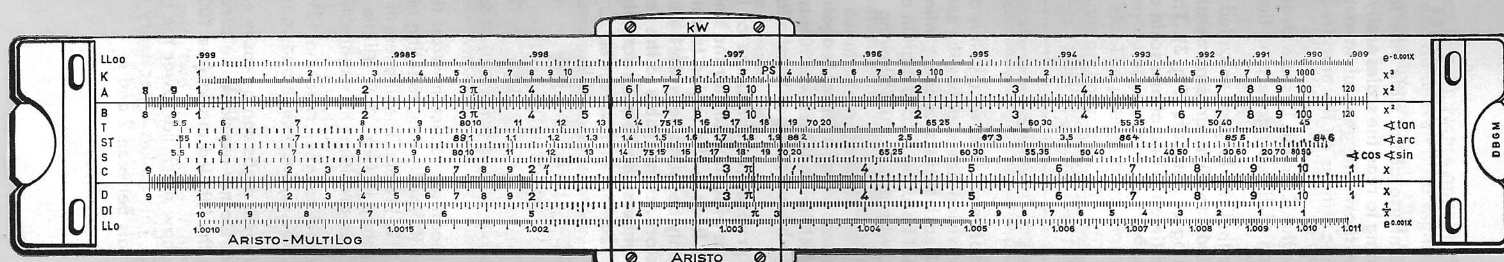

Fig. 2 Back of Rule

2. Reading the Scales

Efficient use of the slide rule for rapid calculations is essentially a matter of learning to read the scales easily and accurately. For guidance refer to Figs. 3—6. They show the general layout of the scales and give examples of several specific settings on the most frequently used scales C and D.

Look the C or the D scale over carefully to get a clear overall picture of the system of division governing the slide rule scales.

Note that the largest, so-called primary intervals are separated by long lines labeled 1 to 10 in large numerals. These represent the first digit in the setting or reading of numbers (Fig. 3). The 10 is always given as 1 as its place is at the same time the end of one scale as well as the beginning of an imaginary second scale.

Fig. 3 The Primary Intervals

Each primary interval again is divided into 10 secondary intervals; each secondary division line supplies the second digit in a number. Between the primary 1 and 2 each of the secondary intervals is labeled with somewhat smaller numerals so that here the second digit in a number can be actually read (1—1, 1—2, 1—3 etc.), whereas in the following ranges from 2 to 10 the second digit has to be counted off.

Each secondary interval is again split up into tertiary intervals separated by the shortest division lines which mark the third digit in a number. Here the beginner has to be watchful, as we shall see.

The progressive shrinkage of the scales makes it necessary to employ three systems of subdivision for the smallest, i. e. the tertiary spaces, to avoid crowding of the lines in the region toward the end. Therefore all tertiary division lines will only be found between 1 and 2. In this first section of the scale the reading is, therefore, comparable to the reading of a rule with metric graduation, so that all numbers can here be actually read to the third digit. (Fig. 4).

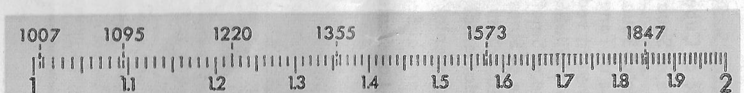

Fig. 4 Reading in the Range 1 to 2

For reading practice now set the cursor hair over the line labeled 1 and advance it, line by line, counting off 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 etc. The fineness of the cursor hairline allows splitting each interval accurately in two. With a little practice even smaller fractions can be located with fair precision by visual estimation.

For practice, after reaching 131, split the interval separating this line from the next, 132, by inspection and count off: 131.0, 131.1, 131.2, 131.3, 131.4, 131.5 etc.

The fourth digits can be easily estimated as the sample settings in Fig. 4 demonstrate. Do not overlook the zero when reading the intervals immediately following the labeled marks (see 1007, 1095, in Fig. 4).

Note how much narrower the intervals have grown as we approach the number 2. This makes it necessary to cut the system of subdivision to two tertiary intervals between 2 and 4.

In this sector only every other interval is marked (Fig. 5). Hence only the third digit of even numbers can be read on the lines and all uneven third digits must be estimated by inspection in the middle of the respective interval. After a little

Fig. 5 Reading in the Range 2 to 4

practical experience you will even be able to estimate the place for the fourth digit fairly accurately.

From this it follows that the even third digits in any number can be counted off on the lines: 200, 202, 204, 206, 208, 210, 212, 214 etc. The uneven third digits are located by interpolation midway between the division lines: 203, 205, 207, 209, 211, 213 etc. Fig. 5 shows several examples in this range (203, 235, 283, 379).

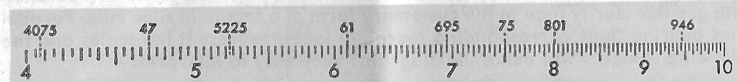

Fig. 6 Reading in the Range 4 to 10

In the range from 4 to 10 the tertiary intervals advance still more rapidly viz. by 5/100th of the primary intervals so that we can count off on the lines: 400, 405, 410, 415, 420, 425, 430 and so on. Intermediate values are located by eye-judgment. Midway between 400 and 405 we can place 4025, a little to the left thereof 402, and a little to the right 403. Conversely the middle of the next interval gives 4075 and so on. Fig. 6 presents a selection of examples.

3. The Principle of Slide Rule Calculation

Mathematical operations on the slide rule are performed by the mechanical addition or subtraction of two segments of graduated scales. This method is best demonstrated by two millimeter scales sliding lengthwise on each other.

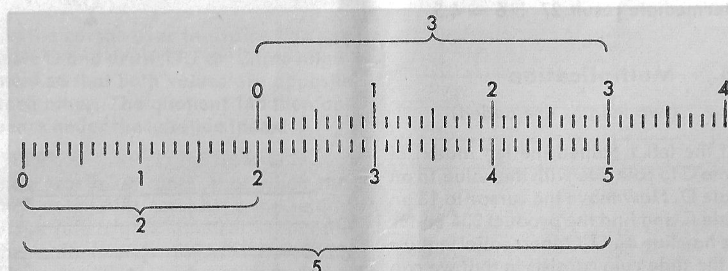

Fig. 7 Graphic Addition by Use of two Ordinary Scales

Fig. 7 shows how a mechanical addition is made with two such scales. When, for instance, the tip 0 of the uppermost scale is moved so as to coincide with the value 2 of the lower scale, we shall find the sum 5 under the value 3 of the upper scale.

Subtraction is the same process in reversed order, i. e. the length 3 of the upper scale is deducted from the total length 5 of the lower scale. It follows that, by simply setting the value 3 over 5, we can read the answer 2 under 0 of the upper scale.

Multiplication and division by use of the slide rule is exactly the same process as that described above, except that we are now dealing with logarithmic lengths, so that by adding or subtracting two segments of line we actually accomplish either a multiplication or a division. In more refined form the above discussed principle of two separate scales is embodied in the slide, movably tongued and grooved to the body of the rule. A cursor is provided to facilitate setting and reading of values to hairbreadth accuracy.

4. Explanation of Working Diagrams used in the Solution of Examples

In the following text an easily memorized method of explanation will be employed, so as to show the step-by-step operations in the respective computation with greater clarity than in the customary form of a facsimile slide rule. Parallel lines bearing their corresponding marginal labels represent the scales and the following symbols will make the diagrams very easy to interpret:

Initial setting
Each subsequent setting
Final result
Setting or reading
of an intermediate result
Reverse the rule
Directions and sequence of movements

Hairline of the cursor

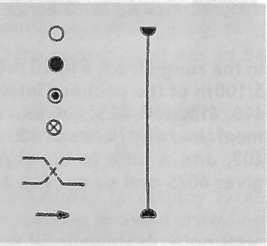

Fig. 8 shows how an example will appear in the diagram.

$$\frac{27}{6} \times 1.7 = 7.65 \text{ (Fig. 8)}$$

Intermediate result $27 \div 6 = 4.5$

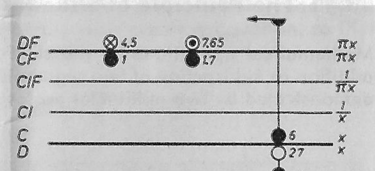

Fig. 8

5. Multiplication

(two lengths are added)

Set the left 1 (called the left index) of scale C to coincide with the value 18 on scale D. Now move the cursor to 13 on scale C and find the product 234 under the hairline on D. One essential feature fo the slide rule consists in that we can perform as many other multiplications by 18 as may be required without changing the initial setting.

Fig. 9 indicates two such further operations, viz.

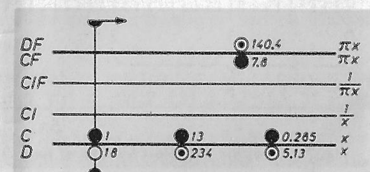

Fig. 9 $18 \times 13 = 234$
$18 \times .285 = 5.13$
$18 \times 7.8 = 140.4$

It will soon be observed, as in 18×7.8, that sometimes the slide projects so far beyond the end of the body scale that no reading can be taken. The simple remedy then consists in setting the right index of the slide over 18 and shifting the cursor to 7.8.

This end-for-end exchange of the indexes is rather troublesome and it can be shown that the upper pair of scales CF and DF offer an easier solution. By studying the rule you will find that the index 1 of scale CF is also matched with the value 18 of the DF scale and, therefore, we can continue reading on these scales where the lower scales break off. In fig. 9 is shown how the example $18 \times 7.8 = 140.4$ can be solved by setting the cursor to 7.8 on CF and reading the result 140.4 on DF. This procedure is always applicable provided that the slide does not project out of the body by more than half its length.

Examples:

$3.04 \times 2.68 = 8.15$	$17.38 \times 4.47 = 77.7$
$3.04 \times 4.05 = 12.31$	$10.03 \times .2484 = 2.49$
$3.04 \times 7.27 = 22.10$	$.739 \times .655 = .484$
$3.04 \times 8.025 = 24.40$	$148.3 \times .039 = 5.78$
$9.81 \times 7.05 = 69.2$	$34.76 \times .0324 = 1.126$
$2.208 \times .0213 = .0470$	$4.33 \times 8.95 = 38.8$

It is often advantageous to make the first setting with the index of the CF scale placed opposite the multiplier on DF, because in this case there is no need to decide whether to start with the right or the left index. Furthermore, in all settings made with the upper pair of scales no more than half a slide length will ever project beyond the body scales. This means that the product can always be read on either the upper or the lower pair of scales, often on both scales simultaneously. It is advisable to repeat the previous exercises by first starting with C and D and next with CF and DF. In this way you can appreciate by experience which type of setting is the better one.

6. Division

(two lengths are subtracted, multiplication in reverse)

Set the cursor over the value 2620 on scale D and draw 17.7 on C into alignment so that both values are opposite each other. The quotient 148 then appears under the left slide index.

Notice:

It is worthy of note that when this manipulation is completed the setting of the rule is also identical to that for the multiplication $148 \times 17.7 = 2620$. The only difference between multiplication and division is the order of the setting and the reading.

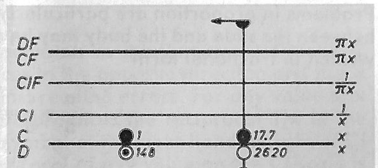

Fig. 10 $2620 \div 17.7 = 148$
Roughly: $3000 \div 20 = 150$

In the problem $582 \div 7.23 = 80.5$ the quotient will be found under the right slide index. It follows that in division no end-for-end switching of the indexes will occur. Later in this text it will be demonstrated how this feature can be usefully employed. The following chapter also contains a reference to the same subject

Examples:

3.78	÷	4.5	=	.840	6.25	÷	.1328	=	47.1
53	÷	7.45	=	7.12	.914	÷	16.29	=	.0561
31.5	÷	22.8	=	1.382	180	÷	π	=	.01745
402	÷	26.66	=	15.08	π	÷	180	=	.01745

The exercises in division, too, should be done, first with the C and D and then with the CF and DF scales. When the problem is set on the upper pair of scales the values are arranged in fractional notation, with the numerator above the denominator, thus:

$$\frac{3.78}{4.5} = .840$$

The answer can be read both over the index of CF on DF and under the right index of C on D.

7. Multiplication and Division Combined

In problems of the type $\frac{a \times b}{c}$ division is usually taken first, because the slide rule is then always ready for the following multiplication.

The intermediate result of the division $345 \div 132 = 2.61$ can be ignored and the cursor moved directly to the value 22 on scale C, opposite which place the final result 57.5 appears on scale D.

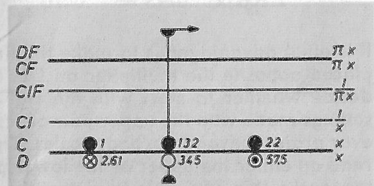

Fig. 11

$$\frac{345 \times 22}{132} = 57.5$$

$$\text{Roughly: } \frac{300 \times 20}{100} = 60$$

8. Proportion

Problems in proportion are particularly easy to solve by slide rule. The juncture between the slide and the body may be regarded as the dividing line in the ratios written in fractional form

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$$

From old habit rule-of-three problems are usually solved as discussed in chapter 7. But the slide rule greatly facilitates and simplifies solution when the problem is changed to proportion form.

The method is probably best shown by an example: 9.5 pounds of a commodity are worth \$ 6.30. How much for 8.4 pounds? In rule-of-three form the solution would be

$$\frac{6.30}{9.50} \times 8.4 = 5.57$$

Stating the same problem in proportion form we write

$$\frac{\text{lb}}{\$} = \frac{9.5}{6.30} = \frac{8.4}{?}$$

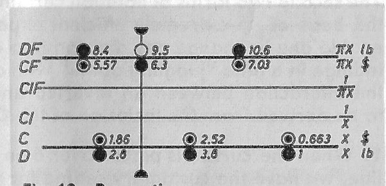

Fig. 12 Proportion

By placing the given weight 9.5 on the DF scale opposite the price 6.30 on CF the ratio of any number on C or CF to the number opposite it on D or DF is constant throughout the entire range of the scales. Accordingly, when the first setting is made as marked by arrowheads in Fig. 12, then, opposite all weights set on DF and D we can find the corresponding prices on CF and C, including the unit price per 1 pound which, in a case like this one, is often of particular interest. Against 8.4 pounds we read the price \$ 5.57 as the answer to the present simple problem. A number of further weight/price relations are marked in the above diagram:

10.6 pounds are worth \$ 7.03 (in scales CF/DF)
3.8 pounds are worth \$ 2.52 (in scales C/D)
2.8 pounds are worth \$ 1.86 (in scales C/D)

An interminable sequence of other ratios can also be instantly read:

$$\frac{\text{lb}}{\$} = \frac{8.4}{5.57} = \frac{10.6}{7.03} = \frac{3.8}{2.52} = \frac{2.8}{1.86} = \frac{1}{0.663} = \ldots$$

Application of the principle of proportion has the important advantage that one is not too strictly bound by inflexible calculating rules as in the previously discussed methods. It makes no difference whether the given ratio is set $\frac{a}{b}$ or $\frac{b}{a}$ as long as the required terms are read in the same vertical order on the scales. In the above example 6.3 might be set on DF against 9.5 on CF and the cursor placed over 8.4 on CF in order to read the answer 5.57 on DF.

Many problems in everyday practice can readily be stated in proportion form for a simplified and more fluent solution by slide rule.

9. The Reciprocal Scales CI and CIF

The CI scale has the same system of subdivision as the fundamental scales C and D, except that the graduation advances in the opposite direction and has a red numeration as a warning signal against reading errors. For any value of x set by cursor on the C scale, the CI scale produces the reciprocal 1/x of the number concerned, as the mathematical symbol at the right-hand margin of CI indicates. Equally important, too, the reciprocal of any value on the CI scale is produced on C. So, for instance, over 5 on C we read 1/5 = 0.2 on CI, while under 4 on CI we find 1/4 = 0.25 on the C scale.

There would be no point in providing a special scale for the sole purpose of determining the reciprocal of a number. Its real usefulness consists of a drastic reduction in slide and cursor movements where complex problems are concerned. Consider that

$4 \div 5$ can be changed to the form $4 \times 1/5$, and
4×5 gives the same as $4 \div 1/5$

The second two forms of notation certainly look a bit unusual but they constitute the basis of an extremely efficient expedient in slide rule work because they enable one to change any division into a multiplication, and vice versa. Let us indulge in a little "juggling game" with oversimplified problems to demonstrate the interaction between an inverted and a normal scale and the advantages to be derived from the inclusion of the CI scale:

1) When the cursor is placed over 6 on D and 2 on C is drawn under the hairline, we have the customary setting for the division $6 \div 2 = 3$. But if we now leave the cursor in place and shift the slide so as to bring the value 2 on the CI scale in line, we perform the multiplication 6×2, wherein the product 12

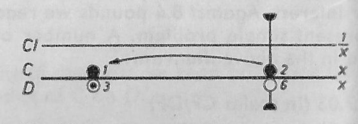

Fig. 13a

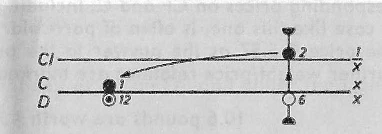

Fig. 13b

appears under the front index of C, exactly as in the customary process of division. As a matter of fact we actually have performed such a division, viz. that of $6 \div 0.5 = 12$ because, simultaneously with the 2 on CI we have automatically also brought the reciprocal value of 2, i. e. 0.5, on the C scale under the cursor hair (multiplication changed to division, Fig. 13a-b).

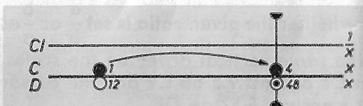

Fig. 14a

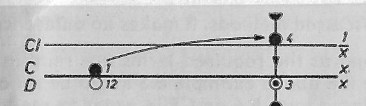

Fig. 14b

2) If we now keep the index of the C scale over 12 on D and shift the cursor to 4 on C, this means that we are multiplying $12 \times 4 = 48$ by the conventional process. But if we set the cursor to 4 on the CI scale instead of C, we find the quotient of the division $12 \div 4 = 3$ on D. This is the explanation: Since under 4 on CI the reciprocal value of 4, viz. $1/4 = 0.25$, is located on C, we have in fact multiplied $12 \times 0.25 = 3$. In other words we have changed a division into a multiplication.

The conclusion to be drawn from the foregoing demonstration is that there are always two kinds of settings, for any division or multiplication, from which a competent slide rule user will always choose the more suitable one in order to maintain the rhythmic alternation between division and multiplication.

The foregoing description of the relationship existing between C and CI also prevails between their folded counterparts CF and CIF. In order to gain a clear understanding of their part in the scale assembly it is worthwhile repeating the above discussed experiments with the group CF/DF/CIF. From studying the processes explained in the foregoing text with due attention, it will now be clear that the inclusion of the CIF scale is the logical final link in the scale system. Slide rule users with experience in the proper use of the folded scales will use the CIF scale as often as CI.

In the process of choosing the proper setting for each successive step in a computation we may freely switch back and forth between C/D/CI and DF/CF/CIF, as witness two examples:

$$\frac{15.3}{2.24} \times \frac{1}{5.3} = 1.29$$

Divide 15.3 by 2.24 with DF/CF. Shift the cursor to 5.3 on CIF. Read the answer 1.29 on DF.

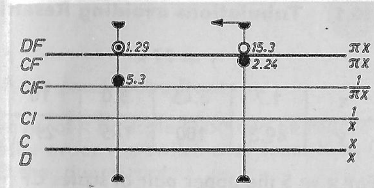

Fig. 15 Computation with CF/DF/CIF

$$1.5 \times 7.9 \times 1.69 = 20$$

Perform the first multiplication 1.5×7.9 in division form with 1.5 on D opposite 7.9 on CI. The third factor can then be set immediately by cursor shift to 1.69 on C, where the final product 20 appears on D.

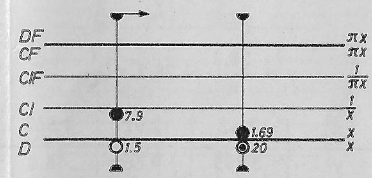

Fig. 16 Computation with C/D/CI

From the foregoing detailed studies these simple rules can be formulated:

1) Whenever an intermediate result in the course of a computation appears under one of the slide indexes and the next step is a multiplication, set by cursor on C or CF; when the next step is a division use CI or CIF.

2) When an intermediate result appears under the cursor hairline on one of the body scales D or DF and the next step is a multiplication, shift the slide so as to bring the multiplier on CI or CIF under the cursor line and read the answer under the respective slide index. If, however, the next step is a division, solve in the customary division form with C or CF in order to get the quotient under a slide index. In either case the slide is correctly adjusted for the next step in the short-cut method of alternate division and multiplication.

10. The Folded Scales CF and DF

We have already had occasion to use these scales in multiplication and division, in which class of computation they are principally employed (See chapters 5 to 9). The folded scales are identical to the fundamental scales, except that any location on one type of scale is laterally displaced by the value $\pi = 3.14159$ relative to the other scale. This means that the value π on the folded scales is exactly matched with the indexes of the fundamental scales. Consequently the graduation runs from π past 1, in about the middle of the rule, to π at the right extreme. There are short extensions of the graduations at both ends for greater convenience.

The index 1 of the CF scale is always aligned to the same value on DF to which the index 1 of C is aligned on D. It follows that we can also begin a calculation with the CF/DF scales and with advantage, too. There is no more need to decide between two indexes for the first setting.

The two scale pairs CF/DF and C/D form a team working in perfect unison. Whenever the point is reached where one pair fails to give the answer, the other pair takes over. No more "resetting the slide".

The yellow strips on the slide serve as a precaution against errors, since the slide scale C moves over D but CF moves under DF.

10.1 Tabulations avoiding Resetting the Slide

$y = 29\,x$

x	1.7	3.45	5.0	10
y	49.3	100	145	290

For x = 5 the upper pair of scales CF and DF provides the answer without resetting the slide.

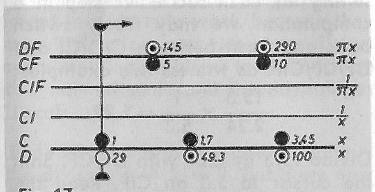

Fig. 17

$$y = \frac{28.2}{x} = 28.2 \times \frac{1}{x}$$

x	7.43	2.92	1.567
y	3.795	9.66	18.0

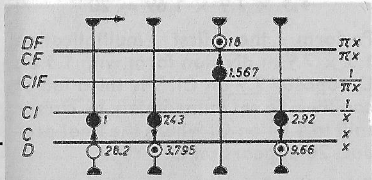

Fig. 18

$$y = \frac{x}{18.2} = \frac{1}{18.2} \times x$$

x	3.17	112.1
y	.1742	6.16

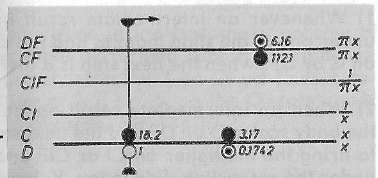

Fig. 19

10.2 Direct Reading of Multiplications and Divisions involving π

Another advantage of this folded arrangement consists in simplifying various computations involving the factor π. It will be clear that any switch-over from D to DF automatically supplies the product of any number set on D multiplied by the factor π. Conversely then the division by π is achieved by following the opposite course.

Typical Problems:

- Circumference of Circles $C = d\pi$
- Angular Velocity $\omega = 2f\pi$
- Area of Circles $A = r^2\pi$

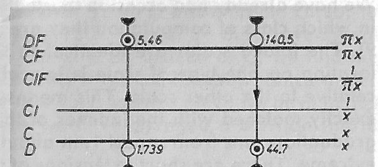

Fig. 20 $x \times \pi$ and $x \div \pi$

The first two formulas can be computed with one cursor setting, whereas we must first perform the multiplication r × r with the scales C and D to find the circle area (see also chap. 16.2).

$1.739\,\pi = 5.46 \qquad \dfrac{140.5}{\pi} = 44.7$

$\dfrac{\pi}{5.73} = .548 \qquad \dfrac{1}{21 \times \pi} = .01516$

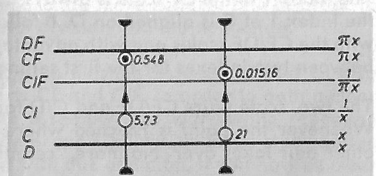

Fig. 21 $\dfrac{\pi}{x}$ and $\dfrac{1}{x \times \pi}$

11. The Scales A, B and K

11.1 Squares and Cubes, Square Roots and Cube Roots

When the cursor hairline is set to any value of x on scale D, x^2 can be read on scale A and x^3 on scale K. In the opposite order the switch-over from K to D furnishes the cube root and from A to D the square root.

Examples:

a) $2^2 = 4 \qquad 2^3 = 8$

b) $3.27^2 = 10.7 \qquad 3.27^3 = 35$

c) $\sqrt[2]{9} = 3 \qquad \sqrt[3]{27} = 3$

d) $\sqrt[2]{51} = 7.14 \qquad \sqrt[3]{364} = 7.14$

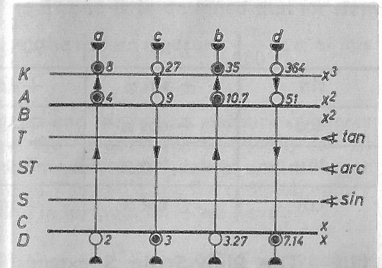

Fig. 22

When it is clear that the root concerned will fall within the range 1 to 10 no special calculating rules need be applied. This is the case when the radicand of a square root lies somewhere between 1 and 100 = 10^2 or the radicand of a cube root between 1 and 1000 = 10^3. In all other cases it is necessary to reduce the radicand to the more handy form of a power of ten notation.

Exercises:

$$\sqrt[2]{3200} = \sqrt[2]{100 \times 32} = \sqrt[2]{10^2 \times 32} = 10 \times \sqrt[2]{32} = 10 \times 5.66 = 56.6$$

$$\sqrt[3]{.1813} = \sqrt[3]{\frac{181.3}{1000}} = \frac{1}{10} \times \sqrt[3]{181.3} = \frac{1}{10} \times 5.66 = .566$$

$$\sqrt[3]{.270} = \sqrt[3]{\frac{270}{1000}} = \frac{1}{10} \times \sqrt[3]{270} = \frac{6.46}{10} = .646$$

$\sqrt[2]{.242} = \dfrac{1}{10} \times \sqrt[2]{24.2} = .492 \qquad \sqrt[3]{1795} = 10 \times \sqrt[3]{1.795} = 12.15$

$\sqrt[2]{3740} = 10 \times \sqrt[2]{37.40} = 61.2 \qquad \sqrt[3]{7.63} = 1.970$

$\sqrt[2]{279} = 10 \times \sqrt[2]{2.79} = 16.70 \qquad \sqrt[3]{.028} = \dfrac{1}{10} \times \sqrt[3]{28} = .304$

$\sqrt[2]{.0948} = \dfrac{1}{10} \times \sqrt[2]{9.48} = .308 \qquad \sqrt[3]{441} = 7.61$

11.2 Multiplication and Division with the A and B Scales

Multiplication and division can also be done by using the scales A and B by the same process as that used for C and D, but the precision obtained will be somewhat less refined. In many problems beginning with a squaring operation it is an advantage to be able to continue the computation on the A and B scales.

It is recommended to repeat the examples given in the chapters 4 to 7 with the scales of squares A and B for practice in the use of these scales and in order to judge the accuracy here obtainable as compared with that of the fundamental scales.

12. Trigonometric Functions

When the hairline is set to the angle on scale S, T or ST, the respective function can be read on the D scale. By reversing the process we obtain the angle corresponding to the given function. Angles are given in degrees divided decimally.

The slide rule can only supply the functions of angles in the first quadrant directly. Use the following table to reduce angles of other quadrants to the first:

	$\pm\alpha$	$90° \pm \alpha$	$180° \pm \alpha$	$270° \pm \alpha$
sin	$\pm \sin\alpha$	$+\cos\alpha$	$\mp \sin\alpha$	$-\cos\alpha$
cos	$+\cos\alpha$	$\mp \sin\alpha$	$-\cos\alpha$	$\pm \sin\alpha$
tan	$\pm \tan\alpha$	$\mp \cot\alpha$	$\pm \tan\alpha$	$\mp \cot\alpha$
cot	$\pm \cot\alpha$	$\mp \tan\alpha$	$\pm \cot\alpha$	$\mp \tan\alpha$

12.1 The Sine Scale S

The Sine Scale S extends from 5.5° to 90° and also contains the cosine values in red numerals, running backwards from right to left. All sines read on the fundamental scales must be given the prefix "0".

Examples: a. sin 30° = .500
b. sin 26° = .438
c. cos 75° = .259
d. cos 42.8° = .733

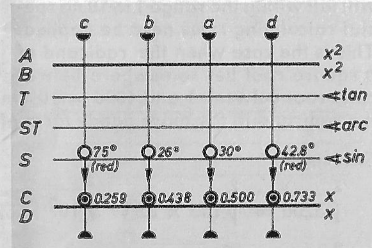

Fig. 23

12.2 The Tangent Scale T

The Tangent Scale T has black numbers for angles from 5.5° to 45° and in opposite progression from 45° to 84.5° in red colour. Since the tangent of any angle $\alpha < 45°$ is always smaller than 1, the corresponding functional values on the D scale take the prefix "0"....

For angles $\alpha > 45°$ apply the red numbers and obtain the function from the red numbers of the CI scale in consideration of $\tan\alpha = 1/\tan(90° - \alpha)$. These values are always larger than 1. With the slide "in neutral", readings can also be taken from scales D or DI. The formula $\cot\alpha = 1/\tan\alpha$ governs the rule for finding the cotangent of an angle.

Therefore read the cotangents on CI for angles $\alpha < 45°$ and on C for angles $\alpha > 45°$ (or on DI and D, respectively).

Examples (see Fig. 24)

a. tan 14° = .249
cot 14° = 4.01
b. tan 81.4° = 6.61
c. cot 68.25° = .399

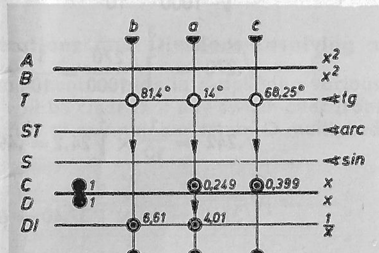

Fig. 24 Slide in "neutral position"

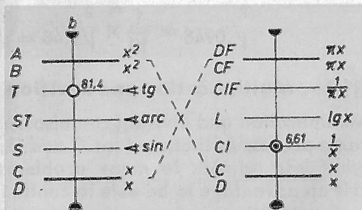

Fig. 25 Slide in operational position, read on rear face

tan 81.4° = 6.61

When the function is given proceed in the reversed order to obtain the angle

arc sin .783 = 51.5° arc tan .764 = 37.4°

arc sin .967 = 75.2° arc cot .983 = 45.5°

12.3 The Scale ST (Small Angles - Large Angles - Degrees ⟷ Radians)

This scale includes angles between 0.55° to 6° and is so designed that for any angle α set on its graduation the C scale gives the radian measure $\frac{\pi}{180} \times \alpha$ of this angle.

For practical purposes the sines, the tangents and the radian measure of angles smaller than 6° are so nearly alike, that the ST scale serves equally for all three functions. Conversely, too, for the cosines and cotangents of angles larger than 84.5° which are not included in the S and T scales, using the complementary angle in this case. Expressing the foregoing in formula form we have:

$$\sin\alpha \approx \tan\alpha \approx \cos(90° - \alpha) \approx \cot(90° - \alpha) \approx \frac{\pi}{180}\alpha = .01745\,\alpha \text{ in radians}$$

Examples: $\sin 1° \approx \tan\ 1° \approx \cos 89° \approx \cot 89° \approx \frac{\pi}{180} = .01745$ rad on C

$$\cot 1° \approx \frac{1}{\tan 1°} \approx \frac{180°}{\pi} = 57.3 \text{ on CI}$$

$$\sin 3.8° = .0663, \quad \cos 88.2° = .0314, \quad \tan 2.74° = .0478.$$

The cosines of small angles and, conversely, the sines of large angles cannot be found directly with the slide rule because the cramped interval between 80° and 90° (0° and 10° in red) of the S scale makes it impossible to place the angle accurately. When these are involved the solution requires the use of the series progression

$$\cos\alpha \approx 1 - \frac{\alpha^2}{2} \quad (\alpha \text{ in radians})$$

$$\cos 1° \approx 1 - \frac{.01745^2}{2} = 1 - .000152 = .999848.$$

Note that the required square of any given angle set on the ST scale is directly readable in radians on the B scale. To find the angle corresponding to a given cosine reverse the process.

12.4 Conversion of Degrees to Radians (and vice versa)

Scale ST is a duplicate of the fundamental scales, modified only in so far as the one graduation is displaced laterally by the value $\frac{\pi}{180}$ relative to the other graduation. Therefore by following the cursor line from scale ST to C we achieve the conversion of degrees to radians, and vice versa. This form of calculation is applicable not only to the small angles discussed above but to large angles as well, by virtue of the decimal subdivision of the degrees, in consideration of the fact that the displacement of the graduations by $\frac{\pi}{180}$ is simply a constant multiplication factor.

Any setting of an angle α may also be regarded as representing $.1\,\alpha$, $10\,\alpha$, $100\,\alpha$ etc. and the decimal point in the radian is then placed accordingly.

For instance: $.1° = .001745$ radians

$10° = .1745$ radians $\tan 89.5° = \frac{1}{\tan .5°} = \frac{1}{.008725} = 114.5$

$100° = 1.745$ radians

When small angles are given in terms of minutes and seconds they can be converted to decimal parts of the degree as follows: $1' = \frac{1°}{60}$ and $1'' = \frac{1°}{3600.}$

12.5 The Gauge Marks for Minutes ′ and Seconds ″

The gauge marks ′ and ″ on the ST scale give a means of direct computation of radian equivalents when the angle is given in minutes or seconds. They represent the conversion factors:

$\frac{180}{\pi} \times 60 = 3438$ for minutes and

$\frac{180}{\pi} \times 60 \times 60 = 206265$ for seconds.

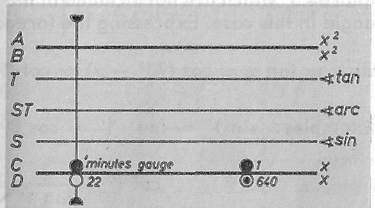

Fig. 26 22′ = .00640 radians

Hence: α in radians $= \frac{\alpha'}{\text{minutes gauge}} = \frac{\alpha''}{\text{seconds gauge}}$

n reversed order: $\alpha' = \alpha$ rad $\times$ minutes gauge ′

$\alpha'' = \alpha$ rad $\times$ seconds gauge ″

Examples:

$22' = \frac{22}{3438} = .00640$ radians $\left(\text{Roughly } \frac{20}{3000} = .006\right)$

$380'' = \frac{380}{206265} = .001843$ radians $\left(\text{Roughly } \frac{400}{200000} = .002\right)$

$.0045$ radians $= .0045 \times 3480 = 15.8'$. (Roughly $.005 \times 3000 = 15$)

The gauge marks for minutes ′ and seconds ″ are a useful aid in solving circle sectors for radii r, arc lengths b and central angles α.

$\alpha = \frac{b}{r} \times$ gauge mark $\qquad b = \frac{\alpha \times r}{\text{gauge mark}}$

Examples:

$\alpha = \frac{.6}{45} \times$ minutes gauge $' = 45.8'$

$b = \frac{48'' \times 67}{\text{seconds gauge}''} = .0156$

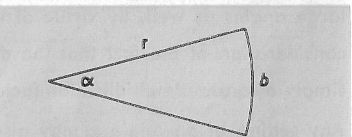

Fig. 27

13. Trigonometric Solution of Plane Triangles

The law of sines is a convincing example of the slide rule's efficiency in solving problems in proportion.

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

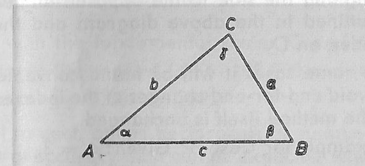

Fig. 28

By setting the given angle on S opposite the given side on D, the other ratios are immediately in coincidence i. e. the angle corresponding to the given side or the side opposite the given angle is directly available.

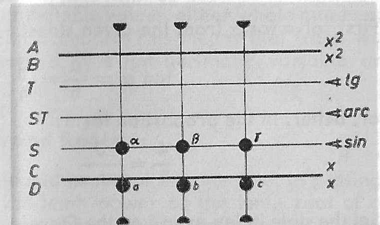

Fig. 29 Setting for the Law of Sines

In practice this form of computation is mostly concerned with right triangles. In this particular case $\gamma = 90°$, hence $\sin\gamma = 1$, and $\alpha = 90° - \beta$, $\beta = 90° - \alpha$. The law of sines is therefore rearranged to the formula.

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{1} = \frac{a}{\cos\beta} = \frac{b}{\cos\alpha}$$

and further: $\tan\alpha = \frac{a}{b}$

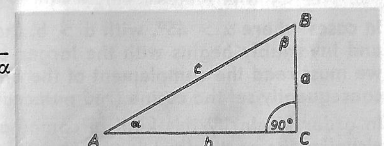

Fig. 30

Depending on the given elements there are two basic operations viz.

A. Given: Any two parts (except case B).

B. Given: The small sides a and b.

Example for Case A:

Given: $c = 5$, $b = 4$

Required: a, α, β

$\frac{5}{1} = \frac{4}{\sin 53.15°} = \frac{3}{\cos 53.15°}$

$\beta = 53.15°$

$\alpha = 90° - 53.15° = 36.85°$

$a = 3$

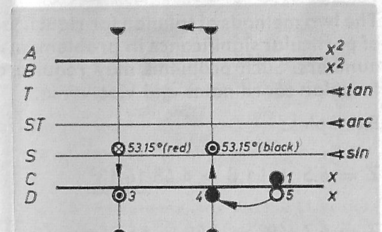

Fig. 31 Given the Hypotenuse

Set the slide index ($\sin 90° = 1$) of the C scale over the hypotenuse 5 on D. The required values a, α and β can now be read by merely moving the cursor. Over the side 4 on the D scale find $\beta = 53.15°$ on S (black numerals). It makes no difference whether the next operation required to find the side 3 on the D scale is the setting of the cursor to $\alpha = 90° - 53.15° = 36.85°$ on scale S with the black numeration, or its direct setting to the cosine 53.15° on the red numeration.

When the given elements are one side and one angle, the first step consists in aligning the side to the opposite angle; the following operations are then as outlined in the above diagram and the hypotenuse is found under the slide index on D.

In some cases it will be found convenient to use the DF scale instead of D to avoid end-for-end changes of the indexes. All sides then appear on the DF scale. The method itself is unchanged.

Example for Case B: Given: $a = 3$, $b = 4$,
To find: c, α, β

First solve for α from the given sides:

$$\tan\alpha = \frac{a}{b} = \frac{3}{4}$$

or, better, in the proportion form:

$$\frac{4}{1} = \frac{3}{\tan\alpha}$$

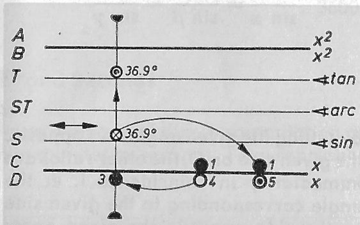

Fig. 32 Given the Two Sides

Set the slide index over 4 on the D scale and move the cursor to 3 on D. Read the angle $\alpha = 36.9°$ by following the hairline to scale T. The next step is to find c by applying the law of sines in $\frac{c}{1} = \frac{3}{\sin 36.9°}$. Since the cursor is already located over the value 3, we now shift the slide so as to bring the angle 36.9° on scale S under the hairline. The answer $c = 5$ can now be read on D under the slide index.

In cases where $\alpha > 45°$, with $a > b$, the computation runs as described above and invariably begins with the larger of the two sides. In this case, however, we must read the complement of the angle (red numerals) on the T scale and consequently set the cosine (red numerals) on S in the next step.

In order to clarify this kind of computation and avoid mistakes it is useful to visualize the particular triangle in your mind's eye.

Examples:	Given the right triangle in which:		Solution:	
	$a = 10.63$	$b = 19.8$	$\alpha = 28.23°$	$c = 22.47$
	$a = 46.7$	$b = 16.5$	$\alpha = 70.53°$	$c = 49.5$
	$c = 78.2$	$\alpha = 22.7°$	$a = 30.2$	$b = 72.3$

The two methods of solution for right triangles discussed in the preceding text are of particular significance in problems involving coordinates, vectors, or complex numbers. Such problems may require conversions of rectangular coordinates to polar coordinates and vice versa.

Examples:

$$Z = 4.5 + j\,1.3 = 4.68\underline{/16.13°}$$

$$Z = 6.7\,\underline{/49°} = 4.39 + j\,5.05$$

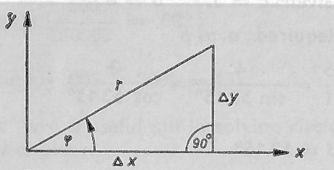

Fig. 33 $\Delta x, \Delta y \longleftrightarrow r, \varphi$

These conversions are very often encountered in electrical engineering. In the component form $Z = a + jb$ the values are easy to add or subtract, whereas the vector form $Z = r\underline{/\varphi}$ is the more suitable for multiplication, division and finding roots and powers.

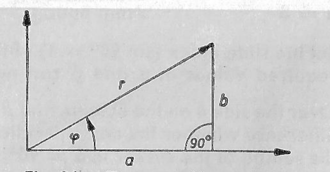

Fig. 34 $Z = a + jb = r\underline{/\varphi}$

14. The Log Log Scales LL0-LL3 and LL00-LL03

All Log Log scales are used together with the fundamental scales C and D. The e^{+x}-scales LL0, LL1, LL2 and LL3 cover the range 1.001 to 100,000 and the e^{-x}-scales LL00, LL01, LL02 and LL03 the range .00001 to .999.

The e^{+x}- and e^{-x}-scales are reciprocal to each other. Reciprocals of numbers < 2.5 can be read with a higher degree of accuracy than can be expected when using the scales CI and CIF, e. g.: the reciprocal value of 1.0170 is .98328.

Attention: The Log Log scales supply immutable values, either whole numbers or numbers with their fractional parts in decimals. This means that when we read 1.35, this is the only value concerned. It is not decimally variable as in dealing with the fundamental scales.

14.1 The 10th, and 1000th Powers and Roots

The Log Log scales are mutually coordinated in such a manner that, in passing from one scale to the adjacent scale, the tenth power or the tenth root of a number set on the one scale can be read on the neighbouring scale, depending on the direction in which the reading is made.

The examples depicted in Fig. 35 will make it clear how the tenth and hundredth power or root of a given number can be determined by the simple process of following the cursor hairline to the appropriate scale. For greater clarity the scales LL00 and LL0 are here shown grouped together with the other Log Log scales on one face of the rule.

Examples:

$$1.015^{.1} = \sqrt[10]{1.015} = 1.001\,49$$

$$1.015^{1} = 1.015$$

$$1.015^{10} = 1.1605$$

$$1.015^{100} = 4.43$$

$$\frac{1}{1.015^{100}} = 1.015^{-100} = .2257$$

$$\frac{1}{1.015^{10}} = 1.015^{-10} = .8617$$

$$\frac{1}{1.015^{1}} = 1.015^{-1} = .985\,22$$

$$\frac{1}{1.015^{.1}} = \frac{1}{\sqrt[10]{1.015}} = 1.015^{-.1} = .99851$$

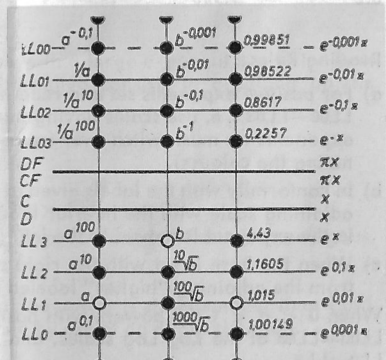

Fig. 35 Reciprocity between the LL scales

Other reading examples based on the range of numbers in Fig. 35.

$\sqrt[10]{4.43} = 1.1605$ $\quad .985\,22^{10} = .8617$ $\quad \sqrt[100]{.2257} = .985\,22$ $\quad 1.001\,49^{1000} = 4.43$

Examples such as the above will hardly ever arise in practice. They are used here to make the system governing the Log Log scales easy to comprehend.

14.2 Powers $y = a^x$

Raising a number to any power is done exactly as multiplications are performed with the fundamental scales.

Procedure:

a) Use the cursor to set the index of scale C to the base "a" on the appropriate LL scale.

b) Shift the cursor hairline to the value of the exponent on C.

c) Read the power y under the hairline on the corresponding LL scale. (See the chapter "Reading Rules".)

When the index of the slide is set to the base "a" we obtain a complete table of values corresponding to the function $y = a^x$. Fig. 36 depicts the setting of the slide to be made for the function $y = 3.2^x$ showing the cursor aligned to the exponent 2.5 and its decimal variates.

Examples:		Reading on scale
$3.2^{2.5}$	$= 18.3$	LL3
$3.2^{.25}$	$= 1.338$	LL2
$3.2^{.025}$	$= 1.02956$	LL1
$3.2^{.0025}$	$= 1.002912$	LL0
$3.2^{-2.5}$	$= .0546$	LL03
$3.2^{-.25}$	$= .7476$	LL02
$3.2^{-.025}$	$= .97134$	LL01
$3.2^{-.0025}$	$= .997096$	LL00
$3.2^{3.1}$	$= 36.8$	LL3
$3.2^{0.36}$	$= 1.520$	LL2

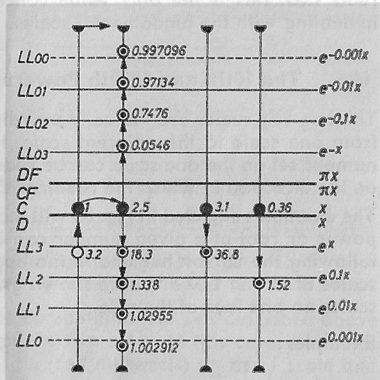

Fig. 36 Powers

Reading Rules:

a) For positive exponents set and read in the same group of scales LL0—LL3 or LL00—LL03 i. e. use scales having numerals of uniform colour. For negative exponents we must switch over from one group of scales to the other (alternating the colours).

b) In conformity with the labels given at the right end of each scale, read on the adjoining scale with the inferior label for each place that the decimal point in the exponent is moved to the left. (Cf. example Fig. 36.)

c) When the base is set with the right slide index, all readings must be taken from the adjoining "higher" labeled scale.

When $0 < a < 1$ the powers with positive exponents are found in the group LL00—LL03 of the Log Log scales, and with negative exponents in the group LL0—LL3.

Examples to clarify the reading rules:

$.685^{2.7} = .36$ $\quad .685^{-2.7} = 2.78$

$1.46^{2.7} = 2.78$ $\quad 1.46^{-2.7} = .36$

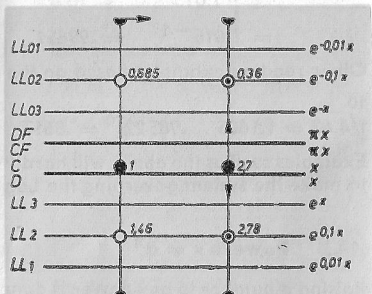

Fig. 37 Left Index of C over Base

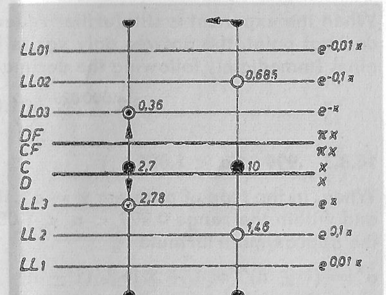

Fig. 38 Right Index of C over Base

Figs. 37 and 38 show the same examples, but in Fig. 37 the slide is set with the left index whereas in Fig. 38 the right index is used.

Examples:			
$1.021^{2.4} = 1.0512$		$3^4 = .0081$	
$1.162^{-4} = .5485$		$3^{-4} = 123$	
$2.13^{5.3} = 55.0$		$.49^{2.8} = .1357$	
$2.13^{.53} = 1.493$		$.49^{1.2} = .425$	
$2.13^{-.53} = .6698$		$.49^{.12} = .918$	
$8.5^{3.7} = 2740$		$.93^{5.1} = 0.691$	
$25.4^{-2.6} = .000223$		$.93^{-5.1} = 1.448$	
$1.0023^{3.7} = 1.00854$		$.9625^9 = .709$	
$e^{-\pi} = .0432$			

14.3 Exceptional Cases of $y = a^x$

Since the range of the Log Log scales is restricted, cases will arise where the exponent is either too great or too small to permit direct reading of the power.

14.3.1 $y > 100{,}000$ and $y < .00001$

When the power corresponding to a base with a large exponent is greater than 100,000 or smaller than .00001 the alternative consists in breaking up the exponent into several factors.

Example:

$$3.14^{19} = 3.14^{6+6+7} = (3.14^6)^2 \times 3.14^7$$
$$= .955^2 \times 10^6 \times 3.02 \times 10^3 = 2.76 \times 10^9$$

For expressions with negative exponents the procedure is, of course, analogous.

14.3.2 $.999 < y < 1.001$

When for a small exponent x, the value of the power is either smaller than 1.001 or greater than .999 the answer is obtained by use of an approximation.

From the series expansion

$$a^{\pm x} = 1 \pm \frac{x}{1!} \log_e a + \frac{x^2}{2!} (\log_e a)^2 \pm \frac{x^3}{3!} (\log_e a)^3 + \ldots$$

can be derived $a^{\pm x} \approx 1 \pm x \log_e a$ for $|x \log_e a| \ll 1$

If the index of C is set opposite the base value a on LL by aid of the cursor line, $\log_e a$ on D (without reading its value) can be multiplied by x. The product added to 1 or, respectively, subtracted from 1 gives the power. The smaller the exponent the more precise will be the result.

Returning to our previous example with the base 3.2 (Fig. 36, we can now continue, e. g.:

$$3.2^{.00025} \approx 1 + .00025 \times \log_e 3.2$$
$$\approx 1 + .0002908 = 1.0002908$$
$$3.2^{-.00025} \approx 1 - .0002908 = .9997092$$

When the exponent is still further reduced through shifts in the location of the decimal point, the answer only varies in respect of the number of ciphers or nines immediately following the decimal point.

$$3.2^{.000\,025} = 1.000\,029\,08$$

14.3.3 $.999 < a < 1.001$

When, in the type of problem $y = a^x$, the value of the base is approximately 1 and within the range $0.999 < a < 1.001$, the solution is worked out by use of the approximate formula

$$a^x = (1 \pm n)^x \approx 1 + x \log_e (1 \pm n) \qquad \log_e (1 \pm n) \approx \pm n \ (\text{for} \ |n| \ll 1)$$

$$(1 \pm n)^x \approx 1 \pm nx \ (\text{for} \ nx \ll 1) \qquad (1 \pm n)^{-x} \approx 1 \mp nx \ (\text{for} \ nx \ll 1)$$

It is immaterial whether, in the range discussed here $\log_e (1 \pm n)$, is set in scale LL or the value n in scale D, in consideration of $\log_e (1 \pm n) \approx \pm n$. The smaller the magnitude of the value n the closer the correctness of the approximation. It follows, then, that where the LL scale breaks off scale D can be used as the continuation of the LL scale, in this case substituting $\pm n$ for $1 \pm n$. When the index of the C scale coincides with n on the D scale this setting is practically identical with the setting $\log_e (1 \pm n)$ within an imaginary additional LL scale covering the range 1.0001 to 1.001 or, respectively, .9990 to .9999 and so on. The computation then continues by looking up the power as previously discussed. Actually any answer read on the D scale is derived from a simple multiplication but has to be complemented by the addition of "1" or the subtraction from "1", as the case may be. When, with growing exponent, the power falls within the readable range of the LL scales readings are taken directly from these scales.

Examples:

$1.000\,23^{3.7} = (1 + .000\,23)^{3.7} = 1.000\,851$
Set on scale D, read on scale D and add 1

$1.000\,23^{37} = 1.008\,55$
Set on scale D, read on scale LLo

$.999\,77^{3.7} = (1 - .000\,23)^{3.7} = .999\,149$
Set on scale D, read on scale D and deduct from 1

$.999\,77^{37} = .991\,54$
Set on scale D, read on scale LLoo

$1.19^{.31}$	$= 1.0554$	(Set on LL2,	read on LL1)
$1.19^{.031}$	$= 1.005\,40$	(Set on LL2,	read on LLo)
$1.19^{.0031}$	$= 1.000\,538$	(Set on LL2,	read on D)
$1.0048^{1.9}$	$= 1.009\,12$	(Set on LLo,	read on LLo)
$1.000\,21^{4.2}$	$= 1.000\,882$	(Set on D,	read on D)
$.999\,79^{4.2}$	$= .999\,118$	(Set on D,	read on D)

14.4 Powers $y = e^x$

When the indexes of the slide and the body scales coincide the rule is adjusted to the equation $y = e^x$. The base $e = 2.718$ on scale LL3 being always aligned with the index of scale D, it follows that any power of e can be found by corresponding movement of the cursor to the exponent on scale D. The setting used in Fig. 35, for instance, would be correct for the exponent 1.489 and its decimal variations:

$$e^{1.489} = 4.43 \qquad e^{.01489} = 1.015$$

$$e^{.1489} = 1.1605 \qquad e^{.001489} = 1.001\,489$$

With the last example we again arrive at the equivalence of $e^x \approx 1 + x$.

14.5 Roots $a = \sqrt[x]{y}$

Expressions containing roots are often easier to handle when changed to the form of a power. In this case set the exponent on the CI scale.

$$\sqrt[3.5]{e} = e^{\frac{1}{3.5}} = 1.3307$$

$$\frac{1}{\sqrt[.35]{e}} = e^{-\frac{1}{.35}} = .0574$$

$$\frac{1}{\sqrt[3.5]{e}} = e^{-\frac{1}{.35}} = .7514$$

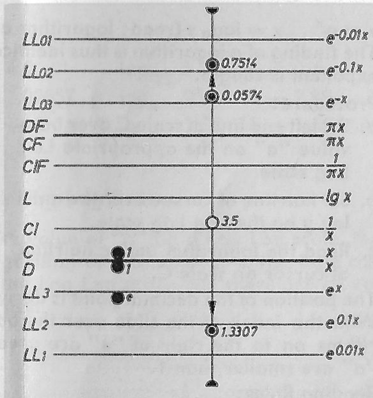

Fig. 39 Solution with the CI Scale

Inversely to the process of raising a number to a power we can also find the roots of numbers by using the Log Log scales in the same manner as the fundamental scales are used in division:

From $y = a^x$ we can derive $\sqrt[x]{y} = a$.

Procedure:

a) Set the radical index on C opposite the radicand y on scale LL.

b) Read the root under either the left or the right slide index on the appropriate Log Log scale.

The reading rules in chap. 14.2 are also in principle applicable in this instance. Bear in mind that when the reading is taken under the right slide index, the answer will appear on the next lower labeled Log Log scale LL1—LL3 or LL01—LL03.

$$\sqrt[.77]{21} = 52.1 \qquad \frac{1}{\sqrt[.77]{21}} = .0192$$

$$\sqrt[7.7]{21} = 1.485 \qquad \frac{1}{\sqrt[7.7]{21}} = .6734$$

$$\sqrt[77]{21} = 1.0403 \qquad \frac{1}{\sqrt[77]{21}} = .96122$$

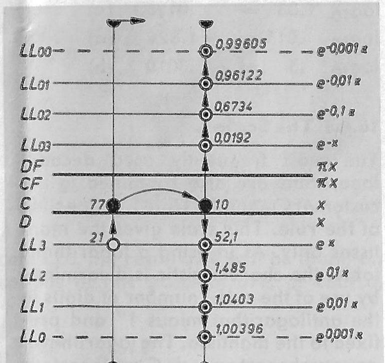

Fig. 40 Roots

14.6 Logarithms

14.6.1 Logarithms to any base

With the Log Log scales logarithms to any base can be determined. By reversing the process of raising a number to a power, we obtain its logarithm:

$y = a^x \quad x = \log_a y$ (read: logarithm of y to the base a).
The finding of a logarithm is thus identical to a problem of powers in which the exponent is sought.

Procedure:

a. Set left end line of scale C over base value "a" on the appropriate Log Log scale.

b. Set hairline of cursor over the antilog y on the Log Log scale.

c. Read the logarithm under hairline of cursor on scale C.

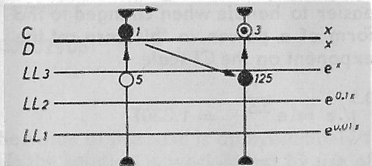

Fig. 41 $\log_5 125 = 3.0$

The position of the decimal point is derived from $\log_a a = 1$.
With the index of the slide over the base "a" on the Log Log scale all logarithms on to the right of "a" are greater than 1 and all values to the left of "a" are smaller than 1.

Reading Rules:

a) Passing from one Log Log scale to the adjacent scale — in the order LL3, LL2, LL1 or LL03, LL02, LL01 — signifies a shift of the decimal point in the logarithm by one place to the left and, in the reverse order, by one place to the right.

b) The logarithms assume positive (negative) values when their antilogs and bases are set on equalcoloured (unequal-coloured) Log Log scales.

When the slide is pushed out to the left of the body, all readings are taken from left of the base value. Since these values are < 1, the decimal point must logically be moved one place to the left.

14.6.2 The decadal logarithms to the base 10 can be found in the same manner by setting the index of scale C to the base 10 on scale LL3 (Fig. 42).

$\log_{10} 50$	=	1.699	(a)
$\log_{10} 2$	=	.3010	(b)
$\log_{10} 1.03$	=	.01283	(c)
$\log_{10} .015$	=	− 1.824	(d)
$\log_{10} .5$	=	− .3010	(b)

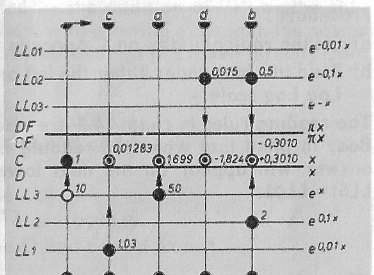

Fig. 42 Decadal Logarithms

14.6.3 The Scale L

The most frequently used decadal logarithms are also contained in the customary mantissa scale L on the slide of the rule. This scale gives the mantissas only. As in using a logarithmic table, the characteristic is determined by use of the rule "number of digits in the antilogarithm minus 1" and prefixed to the mantissa. The logarithm of any number set on scale C can therefore be found by reading the mantissa directly on scale L and, inversely, one can also find the antilogarithm corresponding to a given logarithm.

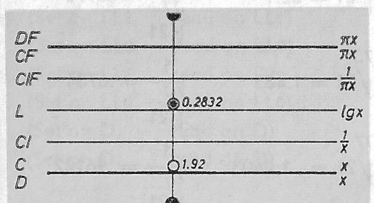

Fig. 43 $\log_{10} 1.92 = .2832$

14.6.4 The natural logarithms to the base e can be read directly on scale D (Fig. 44).

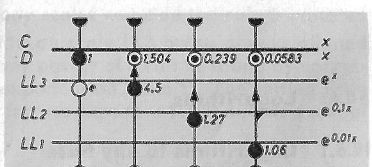

Fig. 44 $\log_e 1.06 = .0583$
$\log_e 1.27 = .239$
$\log_e 4.5 = 1.504$

Examples:

$\log_{10} 6$	= .778	$\log_2 16$	= 4.0	$\log_{.25} 2$	= − .5
$\log_{10} 1.14$	= .0569	$\log_2 1.02$	= .02857	$\log_e .05$	= − 2.994
$\log_{10} 1.015$	= .00647	$\log_2 .25$	= − 2	$\log_e .622$	= − .475

14.7 Solving Proportions with the Log Log Scales

When the slide index is set to some base value of "a" on a Log Log scale, the powers to any exponent and also the logarithm of any number to this base can be obtained. The base a, when set on the Log Log scale, can therefore be regarded as one of the terms in a proportion.

14.7.1

$$y_1 = a^n$$
$$y_2 = a^m$$
$$\log y_1 = n \times \log a$$
$$\log y_2 = m \times \log a$$
$$\frac{\log a}{1} = \frac{\log y_1}{n} = \frac{\log y_2}{m}$$
$$\text{or} \quad \frac{\log_e a}{1} = \frac{\log_e y_1}{n} = \frac{\log_e y_2}{m}$$

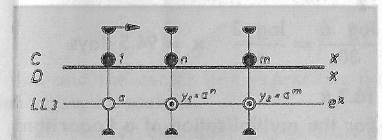

Fig. 45

When three given terms of a proportion are set, the required fourth term can be read and the rule is at the same time adjusted for reading as many other relations proportional to the first ratio as may be required. Here we have another opportunity for applying the principle of proportion for which the slide rule is so eminently well adapted.

14.7.2

$$y = a^{\frac{m}{n}} \qquad \log y = \frac{m}{n} \log a$$
$$\frac{\log y}{m} = \frac{\log a}{n}$$
$$y = 4.3^{\frac{6.8}{2.7}} \qquad \frac{\log y}{6.8} = \frac{\log 4.3}{2.7}$$
$$y = 39.4$$

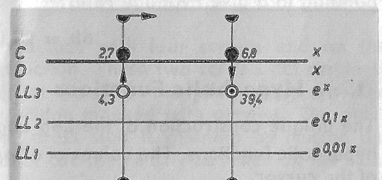

Fig. 46 $\frac{\log 4.3}{2.7} = \frac{\log 39.4}{6.8}$

After setting 2.7 on C opposite 4.3 on scale LL3, the result 39.4 will be found on the LL3 scale under 6.8 of C. Modifications of this problem are, of course, solved analogously:

$$y = \sqrt[2.7]{4.3^{6.8}} \quad \text{or} \quad y^{2.7} = 4.3^{6.8}$$

14.7.3

The formulas of many laws in the natural sciences can be suitably arranged to permit solution in the manner discussed above, when the change in one variable is proportional to the logarithm of the ratio of the other variable.

$$\log \frac{y_2}{y_1} = \text{const} \times (x_2 - x_1)$$

Any change x_1 to x_2 over the increment i entails a change of y_1 to y_2. When the ratio $\frac{y_2}{y_1}$ is given the designation r, i.e. the rest of the original whole quantity, the above equation can be written:

$$\frac{\log r}{i} = \text{const} = \frac{\log r_1}{i_1} = \frac{\log r_2}{i_2} = \ldots$$

Example: Radioactive Decay
A substance is known to disintegrate at the rate of 40% in 30 days, leaving a residue of 60%.
$i_1 = 30$, $r_1 = .6$.
After how many days will 20% be left?
$r_2 = .2$

$$\frac{\log .6}{30} = \frac{\log .2}{x} \qquad x = 94.5 \text{ days}$$

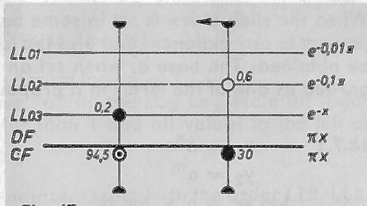

Fig. 47

14.7.4

For the multiplication of a Logarithm by a constant factor the constant on C is set opposite to the base of the logarithm on the Log Log scale. Thus a tabulating position is obtained.

$$x = c \times \log_a y \text{ or } \frac{x}{\log_a y} = \frac{c}{1} = \frac{c}{\log_a a}$$

Examples: $2 \times \log_{10} 100 = 4$
$2 \times \log_{10} 1.8 = .511$

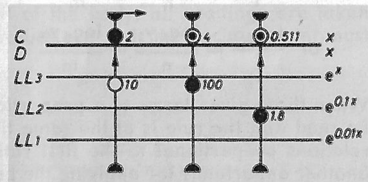

Fig. 48 $x = 2 \times \log_{10} y$

In electrical engineering it is often necessary to compute the decibel corresponding to a given voltage ratio;

$$db = 20 \log \frac{U1}{U2}$$

15. Hyperbolic Functions

The unique construction of the Log Log scale system enables the formation of hyperbolic functions. The values of e^x and e^{-x} can be obtained by one setting of the cursor.

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

16. The Detachable Cursor and its Lines

16.1 The Mark 36

(Applies to 870 and 0970 only)

The front face of the cursor contains a short line to the right of the center line and over the folded scales. The lateral distance of this line from the center line corresponds to the factor 36 for readings on scale DF relative to any setting on the fundamental scale D. By virtue of this arrangement the cursor can be used for conversions of:

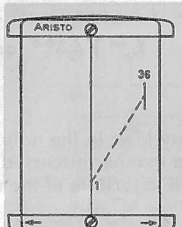

Fig. 49

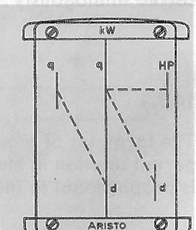

Fig. 50

Hours to Seconds:	1 hour = 3600 seconds
	1 meter per second = 3.6 kilometers per hour
Degrees to Seconds:	1° = 3600″
Years to Days:	1 year = 360 days (for interest computations)

16.2 The Marks for Circle Areas

The intervals between the upper left or the lower right line on the one hand and the center line on the other hand are equal to $\frac{\pi}{4} = .785$ i. e. the factor applicable in computations of circle areas or round sections $A = d^2 \frac{\pi}{4}$ (Fig. 50). To find any required circular area, set the lower right or the center hairline to the given diameter d on scale D and read the area under the center line or the upper left line, respectively, on scale A. With the 20″ model 01070 use the lower right and the upper left hairline.

16.3 The Marks kW and HP

The interval between the upper right line and the center line represents the factor for converting kW to HP, and vice versa, on scale A.
Hence, when the center hairline is set to 20 kW, for example, on the scale of squares, then the upper right line indicates the equivalent in HP viz. 26.8. Inversely, when the short right line is set to 7 HP the center line will produce the equivalent 5.22 kW.
On the 20″ model No. 01070 the kW and the HP marks are attached to the upper left and the upper right cursor line, respectively.

16.4 Detaching the Cursor

The hairlines of the two cursor glasses are precisely matched so that the user can pass from one face of the rule to the other when required in the course of a problem. The accuracy of its adjustment is not disturbed when the cursor is taken off for cleaning.
The cursor glasses are secured, on one face with four screws and on the other with two screws, of special construction. These two screws act similarly to press studs. To remove the cursor from the slide rule, press down with the thumb nails on the end of the cursor bridge marked with arrows. The press studs are thereby released. The upper press stud is opening by raising the free edge of the cursor glass and the cursor can then easily be taken off the rule.

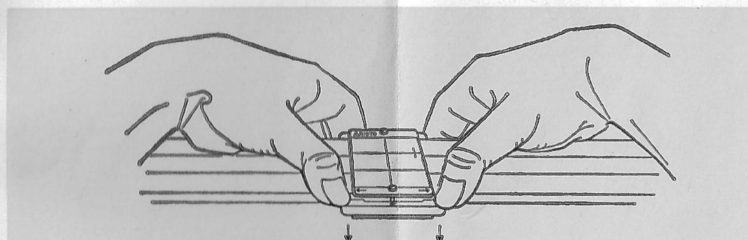
Fig. 51

16.5 Adjustment of the Cursor

Even though the cursor hairs are reliably adjusted, violent jarring of the rule may throw them out of alignment. In such a case loosen the four screws on the cursor face with the HP mark. Turn the slide rule over and shift the other glass until the hairline is accurately aligned to the index lines of the scales. Holding the adjusted glass firmly in position, turn the slide rule over, and adjust the first glass in a similar manner. Tighten all screws carefully to prevent renewed dislocation of the hairlines.

17. Treatment of the ARISTO Slide Rule

The instrument is a valuable calculating aid and deserves careful treatment. Scales and cursor should be protected from dirt and scratches, so that the reading accuracy may not suffer.

It is advisable to give the rule an occasional treatment with the special cleanser fluid DEPAROL followed by a dry polishing. Avoid chemical substances of any description as they are almost certain to spoil the scales.

Do not leave the rule on heated surfaces such as radiators. Do not expose for long periods to strong sunlight. Deformations may occur in temperatures above 140° F (60° C). Rules so damaged will not be exchanged free of charge.